Section-A (Compulsory Question)

1. (*i*) Using definition of limit, prove that :

$$\underset{x \to 1}{\text{Lt}} \frac{x^2 - 1}{x - 1} = 2$$

[**Hint :** Question 3 (*i*) at Page 15 of Spectrum Differential Calculus]

(*ii*) Find the fourth derivative of $x^3 \log x$.

[**Hint :** Question 1 (*d*) at Page 108 of Spectrum Differential Calculus]

(*iii*) Evaluate : $\underset{x \to 1}{\text{Lt}} \frac{\log x}{x - 1}$.

[**Hint :** Question 1 (*v*) at Page 160 of Spectrum Differential Calculus]

(*iv*) State the Taylor's Theorem (with Lagrange's form of Remainder).

[**Hint :** Art-9 at Page 198 of Spectrum Differential Calculus]

(*v*) Examine for concavity and convexity the curve $y = x^3$.

[**Hint :** Question 6 at Page 244 of Spectrum Differential Calculus]

(*vi*) Define oblique asymptote.

[**Hint :** Definition at Page 249 of Spectrum Differential Calculus] (*vii*) What are the necessary and sufficient conditions for any point on the curve to be multiple point ?

[**Hint** : Result of Art-4. at Page 278 of Spectrum Differential Calculus]

(*viii*) If $x = r\cos\theta, y = r\sin\theta$, find $\frac{\partial(x, y)}{\partial(r, \theta)}$.

[**Hint** : Question 4 at Page 427 of Spectrum Differential Calculus]

(8 × 2 ...)

Section-B

Unit-I

2. (*a*) Let : $f(x) \begin{cases} 3 & , \text{ if } x \le 2 \\ ax^2 + bx + 1 & , \text{ if } 2 < x < 3 \\ 7 - ax & , \text{ if } x \ge 3 \end{cases}$

Determine the constants a and b so that $f(x)$ may be continuous for all x.

[**Hint** : Question 33 at Page 74 of Spectrum Differential Calculus]

(*b*) Prove that every real valued function, which is derivable at a point of its domain, is also continuous at that point. Is its converse true ? (6½ ...)

[**Hint** : Art-3 at Page 82 of Spectrum Differential Calculus]

3. (*a*) $\sqrt{x} + \sqrt{y} = \sqrt{a}$, find the value of $\frac{d^2y}{dx^2}$ at $x = a$.

[**Hint** : Example 13 at Page 104 of Spectrum Differential Calculus]

(*b*) If $y = (\sin^{-1} x)^2$, find $y_n(0)$. (7, 6 ...)

[**Hint** : Example 5 at Page 134 of Spectrum Differential Calculus]

Unit-II

4. (*a*) Find the values of constants a and b so that :

$$\underset{x \to 0}{\text{Lt}} \frac{x(1 + a\cos x) - b\sin x}{x^3}$$

exists and equal to 1.

(*b*) State and prove Rolle's Theorem. (6½, 7)

[**Hint :** Art-2 at Page 179 of Spectrum Differential Calculus]

5. (*a*) Use Lagrange's mean value theorem to prove that :

$$1 + x \leq e^x \leq 1 + x e^x \; \forall \, x \geq 0$$

[**Hint :** Example 1 at Page 216 of Spectrum Differential Calculus]

(*b*) Use Maclaurin's theorem with Lagrange's form of remainder to expand sin x as for the nth term in terms of ascending powers of x. (7, 6½)

[**Hint :** Example 8 at Page 209 of Spectrum Differential Calculus]

Unit-III

6. (*a*) Determine the interval in which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards or downwards.

[**Hint :** Question 12 (*i*) at Page 244 of Spectrum Differential Calculus]

(*b*) Prove that the curvature of a circle is constant and is equal to the reciprocal of the radius. (6½, 7)

[**Hint :** Art-3 at Page 317 of Spectrum Differential Calculus]

7. (*a*) Find all the asymptotes of the curve $x^3 + y^3 - 3axy = 0$.

[**Hint :** Example 3 at Page 257 of Spectrum Differential Calculus]

(*b*) Examine the curve $x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0$ for a double point and show that it is a cusp. (7, 6½)

[**Hint :** Example 7 at Page 287 of Spectrum Differential Calculus]

Unit-IV

8. (*a*) Discuss the continuity of the function $f(x, y)$ defined by :

$$f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}} & , \; (x, y) \neq (0, 0) \\ 0 & , \; (x, y) = (0, 0) \end{cases}$$

at origin.

(*b*) If $u = \tan^{-1}\left(\frac{x^2 - y^2}{x + y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\sin 2u$. (7, 6½)

[**Hint** : Question 5. (*i*) at Page 402 of Spectrum Differential Calculus]

9. (*a*) Find the maximum and minimum value of $x^2 + y^2$ subject to the condition $3x^2 + 4xy + 6y^2 = 140$.

[**Hint** : Example 1 at Page 419 of Spectrum Differential Calculus]

(*b*) Show that the functions :

$$u = \frac{x}{y - z}, v = \frac{y}{z - x}, w = \frac{z}{x - y}$$

are not independent of one another. Also find the relation between them.

(7, 6½)

[**Hint** : Example 3 at Page 438 of Spectrum Differential Calculus]

Paper : MATH202TH

Time : 3 Hours] [Maximum Marks : 70

Note : Section-A is compulsory. In Section-B attempt *one* questions each from Units I, II, III and IV.

SECTION—A

(Compulsory Question)

1. Do as directed :

(*i*) Let Q_+ be the set of all positive rational numbers and * a binary operation on Q_+ defined by $a * b = \frac{ab}{3}$. The inverse of 'a' is ....

(*ii*) Define order of an element of a group G.

(*iii*) If H and K are finite subgroups of a group G, then O (HK) = ...

(*iv*) The index of every subgroup of a finite group .... the order of the group.

(v) Every quotient group of cyclic group is cyclic. (True/False)

(vi) State first theorem of Homomorphism.

(vii) Define Integral domain.

(viii) An arbitrary intersection of subrings is .... subring.

$(2 \times 8 = 16)$

Section-B

Unit-I

2. (a) Let Q* denotes the set of all rational numbers except -1. Show that Q* forms an infinite abelian group under the operation * defined by

$a * b = a + b + a\,b,$ for all $a, b \in Q^*$.

(b) Let a, b and x be any elements of a group G. Then prove that :

(i) $O(a^{-1}) = O(a)$ c

(ii) $(x^{-1}\, a\, x)^k = x^{-1}\, a^k\, x$ for all $k \in I$

(iii) $O(a) = O(x^{-1}\, a\, x)$

(7, 6½)

3. (a) Show that the set $\{1, 2, 3, 4, \ldots\ldots\ldots\, p-1\}$, where p is a prime number forms a finite abelian group of order $p - 1$ under the composition of multiplication modulo p

(b) If $a^2\, b = b^2\, a = b$ for all $a, b \in G$ (a semi group), then prove that G is abelian. (7, 6½)

Unit-II

4. (a) A non-empty subset H of a group G is a subgroup of G if and only if

$a \in H, b \in H \Rightarrow a b^{-1} \in H.$

(b) The intersection of an arbitrary collection of subgroups of a group is again a subgroup of the group. (7, 6½)

5. (a) Prove that every subgroup of a cyclic group is cyclic. Is the converse true ?

(b) State and prove Lagrange's theorem. (7, 6½)

Unit-III

6. (a) A subgroup H of a group G is a normal subgroup of G if and only if $g h g^{-1} \in H$ for every $h \in H$ and $g \in G$.

(b) If H is a subgroup of G such that $x^2 \in H$ for all $x \in G$, then prove that H is a normal subgroup of G. (7, 6½)

7. (a) Let N be a normal subgroup of a group G. Show that G/N is abelian if and only if for all $x, y \in G$, $x y x^{-1} y^{-1} \in N$.

(b) The necessary and sufficient condition for a Homomorphism of a group G into a group G′ with kernel K to be isomorphism is that $K = \{e\}$.

(7, 6½)

Unit-IV

8. (*a*) Show that the set $Z_n = \{0, 1, 2, \ldots\ldots n-1\}$ forms a finite commutative ring with unity, under addition and multiplication modulo n. (n is a prime > 1).

   (*b*) Prove that every field is an Integral domain. (7, 6½)

9. (*a*) Prove that intersection of a family of subrings of a ring R is a subring of R.

   (*b*) Prove that the sum and product of two ideals is again an ideal.

(7, 6½)

(Core)

PAPER : MATH201TH

Time : Three Hours | Maximum Marks : 70

Question No. 1 is compulsory. Attempt one question each from Units I, II, III, IV. Marks are given against the questions.

Section-A (Compulsory Question)

(*i*) If a, b, c, d are positive real numbers such that $a < b$ and $c < d$, then $a.c. < b.d.$

[**Hint** : Art-1.9 at Page 10 of Spectrum Real Analysis]

(*ii*) Solve : $2x - 3 < 5x + 3 < 2x + 3$.

[**Hint** : Question 1 (*vii*) at Page 21 of Spectrum Real Analysis]

(*iii*) State Archimedian property of real numbers.

[**Hint** : Statement of Art-1.25 at Page 47 of Spectrum Real Analysis]

(*iv*) State bounded and unbounded sequences.

[**Hint** : Art-2.2 at Page 71 of Spectrum Real Analysis]

(*v*) State Cauchy's second theorem on limits.

[**Hint** : Statement of Art-2.21 at Page 126 of Spectrum Real Analysis]

(*vi*) Show that the series $1^2 + 2^2 + 3^2 ..... + n^2 + ...$ diverges to ∞.

(*vii*) Find the radius of convergence of the power series $\sum \left(1+\frac{1}{n}\right)^{n^2} x^n$.

[**Hint** : Example 4 at Page 306 of Spectrum Real Analysis]

(*viii*) State M_n – test for uniform convergence.

[**Hint** : Statement of Art-4.7 at Page 256 of Spectrum Real Analysis]

(2 × 8 = 16)

Section-B

Unit-I

2. (*a*) Show that $\sqrt{7}$ is not a rational number.

[**Hint** : Question 3 at Page 4 of Spectrum Real Analysis]

(*b*) Solve : $\frac{7x+5}{x-3} < 2$. (6½, 7)

[**Hint** : Question 3 (*i*) at Page 21 of Spectrum Real Analysis]

3. (*a*) For what values of x is $x^2 - x - 30 > 0$?

[**Hint** : Question 5 (*i*) at Page 41 of Spectrum Real Analysis]

(*b*) Find g.l.b. and l.u.b. for $\{\sin^2 x + \cos^4 x : x \in R\}$. (6½, 7)

[**Hint** : Example 5 (*i*) at Page 52 of Spectrum Real Analysis]

Unit-II

4. (*a*) Prove that : $\underset{n \to \infty}{\text{Lt}} \sqrt[n]{a} = 1 \;\forall\; a > 0$.

[**Hint** : Art-2.17 (*ii*) at Page 98 of Spectrum Real Analysis]

(*b*) State and prove Cauchy's first theorem on limits. (6½, 7)

5. (*a*) Prove that $\lim_{n \to \infty} \left[\frac{1}{(n+1)^2} + \frac{1}{(n+2)^2} + \ldots\ldots + \frac{1}{(n+n)^2} \right] = 0$.

[**Hint** : Question 2 at Page 105 of Spectrum Real Analysis]

(*b*) Show that the sequence $\left\{ \frac{2n-9}{3n+1} \right\}$ is monotonically increasing. (6½, 7)

[**Hint** : Question 2 (*i*) at Page 121 of Spectrum Real Analysis]

Unit-III

6. (*a*) Show that $\sum (-1)^{n-1} \frac{n}{n+1}$ oscillates finitely.

[**Hint** : Question 4 (*i*) at Page 180 of Spectrum Real Analysis]

(*b*) Discuss the convergence or divergence of the series $\sum \frac{1}{n(n+1)}$. (6½, 7)

[**Hint** : Question 3 (*iv*) at Page 197 of Spectrum Real Analysis]

7. (*a*) Discuss the convergence of the series $\sum \left(\sqrt[3]{n^2+1} - \sqrt[3]{n^2-1} \right)$.

[**Hint** : Question 7 (*ii*) at Page 198 of Spectrum Real Analysis]

(*b*) Discuss the divergence of the series :

$$\frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \ldots\ldots$$ (6½, 7)

[**Hint** : Question 1 at Page 243 of Spectrum Real Analysis]

Unit-IV

8. (*a*) Show that the sequence $\{f_n\}$ where :

$$f_n(x) = \frac{x}{1+nx^2}, x \in R$$

converges uniformly on any closed interval.

PAPER : MATH310TH

Time : 3 Hours | Maximum Marks : 70

Note : Attempt *five* questions in all, choosing *one* question from each of the Units I, II, III and IV in Section-B. Section-A is compulsory.

Section-A (Compulsory Question)

1. (*i*) Define vector product of four vectors.

[**Hint** : Art-12 at Page 41 of Spectrum Vector Calculus]

(*ii*) What is the volume of a tetrahedron whose coterminous edges $\vec{a}, \vec{b}$ and $\vec{c}$?

[**Hint** : Art-8 (*a*) at Page 25 of Spectrum Vector Calculus]

(*iii*) Prove that $\text{div}\ \vec{r} = 3$.

[**Hint** : Example 1 (*i*) at Page 113 of Spectrum Vector Calculus]

(*iv*) Define solenoidal vector.

[**Hint** : Definition at Page 111 of Spectrum Vector Calculus]

(*v*) Find the value of x, y, z if P (ρ, θ, z) are the polar cylindrical co-ordinates of P (x, y, z).

[**Hint** : $x = \rho \cos \theta,\ y = \rho \sin \theta,\ z = z$]

(*vi*) Explain the term curvilinear co-ordinates of a point.

1. (*i*) Define Triangular Matrix and its types.

[**Hint :** Definition at Page 3 of Spectrum Matrices]

(*ii*) Examine the consistency of the system of equations :

$$x + 2y = 2$$
$$2x + 3y = 3$$

[**Hint :** Question 1 (*i*) at Page 54 of Spectrum Matrices]

(*iii*) Determine the rank of the matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \end{bmatrix}$$

[**Hint :** Question 5 at Page 89 of Spectrum Matrices]

(*iv*) By using elementary row transformation, find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$$

(v) Define characteristic equation of a matrix.

[Hint : Definition at Page 184 of Spectrum Matrices]

(vi) Define coordinates of a vector relative to the basis of a vector space.

[Hint : Definition at Page 274 of Spectrum Matrices]

(vii) Define Translation Mapping.

[Hint : Definition at Page 291 of Spectrum Matrices]

(viii) Find the matrix representation for the projection

$$T: R^2 \rightarrow R^2 \text{ defined by } T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ 0 \end{bmatrix}$$

[Hint : Art-8.10 (*i*) at Page 314 of Spectrum Matrices]

$(8 \times 2 = 16)$

Section-B

(Unit-I)

2. (*a*) Show that every square matrix can be expressed in one and only one way as a sum of a Hermitian and a Skew-Hermitian matrix.

[Hint : Example 3 at Page 20 of Spectrum Matrices]

(*b*) Find the rank of the matrix :

$$A = \begin{bmatrix} 0 & 6 & 6 & 1 \\ -8 & 7 & 2 & 3 \\ -2 & 3 & 0 & 1 \\ -3 & 2 & 1 & 1 \end{bmatrix}$$

(6½, 7)

PAPER : MATH317th

Time : 3 Hours] [Maximum Marks : 70

Note : In Section-A all questions are compulsory. In Section-B each Unit contains two questions and attempt *one* question from each of these Units.

Section-A

(Compulsory Question)

1. (*i*) Define Basic Feasible Solution (BFS).

(*ii*) Define Balanced Transportation Problem.

(*iii*) Explain how an unbalanced transportation problem can be converted into balanced transportation problem.

(*iv*) Write a brief note on VAM to solving transportation problem.

(*v*) What is an assignment problem ?

(*vi*) Explain the difference between a transportation and an assignment problem.

(*vii*) What is two-person zero sum game ?

(*viii*) What do you mean by dominance principle ?

$(2 \times 8 = 16)$

Section-B

Unit-I

2. (*a*) Find the IFS of the transportation problem with the help of North-West corner method :

Market

	A	B	C	Capacity
X	11	21	16	14
Y	7	17	13	26
Z	11	23	21	36
Requirement	18	28	25	

(*b*) Describe the transportation problem and give its mathematical model.

$(7, 6\frac{1}{2})$

3. (*a*) Find out the initial basic feasible solution by applying least cost method :

	1	2	3	4	Supply
1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
Demand	4	16	15	15	

(*b*) With reference to the transportation problem define :

(*i*) Feasible solution

(*ii*) Optimal solution

(*iii*) Non-degenerate basic feasible solution

(*iv*) Degenerate basic feasible solution

(7, 6½)

Unit-II

4. (*a*) Solve the transportation problem for IFS by Vogel's approximation method :

	C_1	C_2	C_3	C_4	Supply
W_1	14	28	20	16	60
W_2	14	24	22	12	60
W_3	10	30	16	18	60
Demand	40	50	40	60	180 / 190

(*b*) Explain the technique used for solving a transportation problem and testing its optimality.

(7, 6½)

5. (*a*) Solve the transportation problem to maximize profit :

Origin \ Destination	1	2	3	4	Supply
A	40	25	22	33	200
B	44	35	30	30	60
C	38	38	28	30	140
Demand	80	40	120	60	

(*b*) Explain Vogel's approximation method to find initial basic feasible solution in transportation problem.

(7, 6½)

(*a*) Solve the game by dominance method :

		Player B		
		I	II	III
Player A	I	1	7	2
	II	6	2	7
	III	5	2	6

(*b*) Solve the game by using graphic method whose payoff matrix is given :

	B			
	I	II	III	IV
A	1	4	–2	–3
	2	1	4	5